Error Propagation Suppression in Self-servo Track Writer by Time-domain Control Design

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Abstract—Control design of Self-servo Track Writer (SSTW) has become an important issue in Hard Disk Drive research. This paper discusses the error propagation problem in SSTW control.

Although the Iterative Learning Control (ILC) has been suggested as a solution to suppress the error propagation in SSTW, existing suggestions design controllers in the iteration domain requiring considerable computation and complicated optimization algorithm. For this reason, this paper suggests SSTW control design in the time domain.

First, reference correction to suppress the error propagation is suggested based on the error propagation study and a novel reference correction is suggested to control the amount of the converged error.

Then, a state space approach is suggested developing a Kalman filter to estimate the absolute head position. The state space based design is extended and provides a formulation for a more general time-domain design of SSTW control.

Index Terms—Hard Disk Drive, Self Servo Track Writer, Error propagation, Kalman filter, Loop shaping

I. INTRODUCTION

A. Necessity of Self Servo Track Writer

Hard-disk Drives are, recently, used not only for the computer system but also for consumer electronics such as hard-disk recorder and car navigation system. This requests higher track density which leads to larger time to write the servo tracks. This increase in the number of servo tracks also cost an enormous investment to secure sufficient facilities such as clean room spaces. This situation forces manufactures to develop novel servo track writing (STW) technologies such as non-contact STW, media STW, self STW, and pattern printing STW.

Among these technologies, self servo track writer (SSTW) uses the read and write elements which is already installed in the head. This makes the servo track writing process be achieved without an external servowriting machine and consequently reduces time and cost [1], [9].

The SSTW uses the offset between the read and write elements of the head. At first, guide servo track patterns are written on a disc drive, then a new servo track is written with the write elements of the head while the read elements reads the previously-written servo track. This is self-propagating of servo track writing.

This self-propagation, however, has several drawbacks. Among them, the most important problem is error propagation. If there is some error in servo tracks, for example, a written servo track cannot be written in a perfect circular form; the error also will be propagated through the self servo track writing process.

The causes of this problem can be listed as follows ([2], [3]):
1) High gain greater than unity in the complimentary sensitivity function especially in high frequency band
2) Disturbances and noises in hard disk drive system
3) Limited measurable output signal

B. Main purpose of this research

It is true that there are several other problems in SSTW other than this error propagation problem, only the error propagation problem, however, is focused on in this paper.

For this problem, we have suggested a control algorithm for SSTW using reference correction and position estimation [4]. Another approach such as iterative learning control has also been applied to this problem [5]. This paper focuses on the characteristic of the proposed algorithm as a design in the time domain and discusses the difference of the proposed SSTW control design from the iterative learning design.

This paper is organized as follows.

At first, the dynamics of the SSTW is modeled and based on that dynamics, the error propagation characteristic is analyzed in Session II. Then Session III analyzes the propagation characteristics in frequency domain, and reflect in correction of reference for the next servowriting. In Session IV, we design a reference correction method focused on the time domain characteristics, not iteration domain. Simulation results verify our analysis and design method.

Lastly, in Session V, a method to depress the error propagation is proposed using estimation of the absolute head position. Kalman filter is adopted and shows some improvements comparing with the conventional method. This Kalman filter design can be a more generalized design of SSTW control in the time domain.

II. MODELING OF SELF SERVO TRACK WRITER AND FORMULIZATION OF THE PROBLEM

As explained in the previous session, the SSTW uses the writing element and read element in the head; it writes new servo track, tracking the previous servo track signals and repeats the same process. Since the SSTW tracks previous servo tracks when it writes new track, the control for the SSTW can be the same with the conventional following control.
A. Modeling of SSTW

Figure 1 shows the propagation characteristics in the SSTW as a block diagram.

\[
\begin{align*}
    y_{n-1} & \rightarrow \xi_n \rightarrow C \rightarrow d_n \rightarrow P \rightarrow r_n \rightarrow y_n \\
    \xi_{n+1} & \rightarrow d_{n+1} \rightarrow C \rightarrow P \rightarrow r_{n+1} \rightarrow y_{n+1}
\end{align*}
\]

Fig. 1. Propagation in Self-servo Track Writer

One loop consisting of a plant \( P \) and a controller \( C \) represents the following controlled system loop. All the symbols used in figures and remainder of this paper are explained in Table I.

<table>
<thead>
<tr>
<th>TABLE I</th>
</tr>
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<tbody>
<tr>
<td>DEFINITION OF SYMBOLS</td>
</tr>
<tr>
<td>( y )</td>
</tr>
<tr>
<td>( \Delta y )</td>
</tr>
<tr>
<td>( \xi )</td>
</tr>
<tr>
<td>( u^{ff} )</td>
</tr>
<tr>
<td>( d )</td>
</tr>
<tr>
<td>( r )</td>
</tr>
<tr>
<td>( N )</td>
</tr>
<tr>
<td>( e )</td>
</tr>
<tr>
<td>( E^s )</td>
</tr>
<tr>
<td>( E^r )</td>
</tr>
<tr>
<td>( r_i ) (subscript)</td>
</tr>
<tr>
<td>( KF ) (superscript)</td>
</tr>
</tbody>
</table>

Note that the offset between write element and read element in the head in Figure 1 is assumed to be zero in the remainder of this paper to simplify the analysis.

The absolute head position \( y_n \) in \( n \)th track is given to \((n+1)\)th track as a position reference; this is the propagation characteristic is the SSTW.

B. Error Propagation in SSTW

Due to the propagation characteristics in the SSTW, errors caused by mechanical disturbances in one track are reproduced from one track to the next. Although the feedback controller (\( C \) in Figure 1) suppresses those disturbances it cannot eliminate the disturbances perfectly. The output error caused by the unsuppressed disturbances will be propagated or even amplified as the track number increases.

C. Simulation using Benchmark Problem

This paper adopts a benchmark problem software ([6]) to analyze and simulate the proposed control algorithm. This benchmark software has been developed by a working group in the Institute of Electrical Engineers of Japan -Technical Committee for Mass Storage Servo Control.

III. REFERENCE CORRECTION DESIGN BASED ON THE FREQUENCY CHARACTERISTICS OF ERROR PROPAGATION

In this section, reference correction design is discussed. This approach is different from the estimation of the head position and also can be combined with the head position estimation suggested in the previous session. [8] is one of these approaches. This approach can also be interpreted as iterative learning control or repetitive control ([10]) as it uses the signals in the previous execution.

A. Derivation of Propagation Characteristic Function

Figure 2 shows the block diagram of the reference correction in SSTW. Measured data in the previous track is filtered and provided as a reference correction in the next track.

\[
\begin{align*}
    y_{n-1} & \rightarrow \xi_n \rightarrow u^{ff}_n \rightarrow C \rightarrow d_n \rightarrow P \rightarrow r_n \rightarrow y_n \\
    \xi_{n+1} & \rightarrow d_{n+1} \rightarrow C \rightarrow P \rightarrow r_{n+1} \rightarrow y_{n+1}
\end{align*}
\]

Fig. 2. SSTW Control using Reference Correction

It is another interesting point that the estimation of head position by accumulating the PES can be categorized as this reference correction which sets \( F \) to 1 or 0.88.

The transition matrix on how two variables \( u^{ff} \) and \( y \) propagate is calculated as Equation (1).

\[
\begin{align*}
    \begin{pmatrix}
        u^{ff}_{n+1}[k] \\
        y_n[k]
    \end{pmatrix}
    &=
    \begin{pmatrix}
        F(1-T) & F(1-T) \\
        T & T
    \end{pmatrix}
    \begin{pmatrix}
        u^{ff}_n[k] \\
        y_{n-1}[k]
    \end{pmatrix}
\end{align*}
\]

where \( T \) represents the complementary sensitivity function \( \frac{PC}{1+PC} \).

From this matrix, the relationship between \( u^{ff}_n \) and \( y_{n-1} \) is given as

\[
T_{u^{ff}}[k] = F(1-T)y_{n-1}[k].
\]

This simplifies the propagation characteristics as the below equation.

\[
y_{n+1} = (T + F - TF)y_n
\]

(3)

Error propagation can be suppressed by shaping this propagation characteristics; \( T + F - TF \). Using the descriptions \( C, P \) in Figure 2, the propagation characteristics can be described as

\[
T + F(1-T) = \frac{CP}{1+CP} + \frac{F}{1+CP} = T_{pr}.
\]

(4)

The reference correction filter \( F \) can be designed based on this equation. If \( F \) is designed to make the infinity norm of \( T_{pr} \) less than unity, error propagation will be converged to zero, which is proved in the iterative learning control theory [11].
B. Analysis of Propagation Characteristics of Track Errors

The disturbance and noise described in HDD - torque disturbance, flutter noise, RRO noise and sensor noise - are considered in the propagation analysis. In the remainder of this paper, all noises except the sensor noise are dealt with as one equivalent noise which is applied to the output of the plant and described as \( o \) while the sensor noise is described as \( \xi \).

The absolute track squeeze error, \( E^a \) can be given as \( -y[k] \) since the reference position for each track is assumed to be zero. Its propagation is described as Equation (5). Equation (5) is the case of the relative track squeeze error \( E^r \).

\[
E^r_{n+1}[k] = T_{pr} E^r_n[k] - FS_o[k] + S o_{n+1}[k] + T\xi_{n+1}[k] \\
E^a_{n+1}[k] = T_{pr} E^a_n[k] - F_S (o_{n+1}[k] - o_n[k]) + S(o_n[k] - o_{n+1}[k]) + T(\xi_n[k] - \xi_{n+1}[k]), \tag{5}
\]

where \( S \) represents the sensitivity function (1-T).

The absolute squeeze error is excited by \( o \) and \( \xi \) through \((1 - F)\) and \( T \) respectively. Note that \( F \) will affect not only the propagation characteristics \( T_{pr} \) but also the way the noise \( o \) excites the absolute track squeeze error.

Comparing Equation (5), the relative track error is excited by the difference of the noise between the previous track and the current track.

IV. SIMULATION OF PROPOSED REFERENCE CORRECTION USING BENCHMARK PROBLEM

A. Convergence of Error Propagation

Based on Equation (4), \( F \) is designed as Equation (6) to decrease the norm of \( T_{pr} \) under unity, with the gain \( K \) between 0 and 1;

\[
F = K + (K-1)CP_n, \quad (6) \\
T_{pr} = T + F(1-T) = K \quad (7)
\]

where \( P_n \) is the nominal dynamics of VCM which consists of a second order system. This \( F \) makes the propagation characteristic \( T_{pr} \) in Equation (4) \( K \) if \( P_n = P \) this is the same idea which has been suggested in a paper [12].

Simulations are done changing \( K \) to 0.2, 0.5, 0.8. The results are illustrated in Figure 3.

![Fig. 3. 3σ of Errors and Feedforward Input](image)

Simulations are conducted until the track reaches 100th track. Both the relative track error and absolute track error do not diverge and converge to particular values for \( K \) values.

In ideal case where \( P = P_n \), \( K \) will be the propagation gain which means the smaller \( K \) is, the faster the values of PES and \( Y \) will converge. This is verified in Figure 3(b). However, the small \( K \) which will improve the convergence performance leaves large converged errors.

Large gain \( K \) can decrease these converged error values; as the gain \( K \) approaches near 1, the level of the residual errors decreases. Finally when the gain \( K \) becomes 1, the stability limitation of propagation characteristic in Equation (6), the errors starts to diverge.

Another noticeable point in this simulation is the difference between converged values of two errors. In the case of Gain 0.9, the relative track error converges around 0.14 while the absolute track error converges around 0.36. Relative track error is more important in terms of practical use. If the distance between adjacent tracks is constant with small deviation, it can be used as servo track although the absolute shape of the track is not a perfect circle.

B. Control of Converged Error

This simulation verifies the suggested control design can control the convergence performance. The problem is the amount of the converged errors. This amount of the converged errors is related with the term \( -FS_o[k] \) in Equation (5).

Since \( F \) is determined as Equation (6) to decide the convergence, another degree of freedom is necessary so as to change the amplitude of the converged error. To this end, information of the \((n - 1)\)th track is utilized in the reference correction in the \((n+1)\)th track. Figure 4 is the illustration of this new correction.

![Fig. 4. SSTW Control using Reference Correction](image)

In order to make sure that the added \( F_2 \) can decrease the converged error, a simple controller of \( F_1 \) and \( F_2 \) is designed like the following:

\[
F_1 = K + (K-1)CP_n, \quad (8) \\
F_2 = -RF_1 \quad (9)
\]

Simulation is done with this control design, and the results are shown in Figure 5.
With $R = 0$, the result is same with the previous simulation result without $F_2$ correction. Figure 5 (b) verifies that the converged absolute error can be changed by $F_2$. However, the decrease in the error is not large, and more detailed control of $F_2$ is necessary.

In order to design $F_2$ in a more detailed way, this paper studies the dynamics of SSTW under $F_2$ correction. With the $F_2$ reference correction, the control input $u_{nf}^{ff}$ and the head position $y$ are determined like the following:

$$T_{ff}^{n+1} = F_1S(y_n - a_n) + F_2S(y_{n-1} - a_{n-1})$$  \hspace{1cm} (10)

$$y_{n+1} = (T + F_1S)y_n + F_2Sy_{n-1} - F_1So_n - F_2So_{n-1} + Tξ_{n+1} + So_{n+1}$$  \hspace{1cm} (11)

Since $y_{n+1}$ is affected by both $y_n$ and $y_{n-1}$, the system should be described using information of two tracks.

$$
\begin{pmatrix}
  y_{n+1} \\
  y_{n+2}
\end{pmatrix} =
\begin{pmatrix}
  T_{pr2} & T_{pr1} \\
  T_{pr1}T_{pr1} + T_{pr2}
\end{pmatrix}
\begin{pmatrix}
  y_{n+1} \\
  y_{n+2}
\end{pmatrix}
+\begin{pmatrix}
  -F_2S \\
  -T_{pr1}F_2S - T_{pr1}F_1S - F_2S
\end{pmatrix}
\begin{pmatrix}
  a_{n-1} \\
  a_n
\end{pmatrix}
+\begin{pmatrix}
  S \\
  0
\end{pmatrix}
\begin{pmatrix}
  T_{pr1}S - F_1S \\
  0
\end{pmatrix}
\begin{pmatrix}
  a_{n+1} \\
  a_{n+2}
\end{pmatrix}
+\begin{pmatrix}
  T \\
  0
\end{pmatrix}
\begin{pmatrix}
  ξ_{n+1} \\
  ξ_{n+2}
\end{pmatrix},
$$  \hspace{1cm} (12)

where $T_{pr1}$ represents $T + F_1S$, and $T_{pr2}$ represents $F_2S$ from Equation (11).

The matrix

$$\begin{pmatrix}
  T_{pr2} \\
  T_{pr1}T_{pr1} + T_{pr2}
\end{pmatrix}
$$

decides the convergence characteristic and

$$\begin{pmatrix}
  -F_2S \\
  -T_{pr1}F_2S - T_{pr1}F_1S - F_2S
\end{pmatrix}
$$

decides the amount of the converged error. Control design based on this consideration is our future work.

V. INTRODUCTION OF GENERAL SSTW CONTROL DESIGN USING STATE SPACE DESCRIPTION

In order to design controller $F$ in a more systematic way, a hint for generalized time-domain SSTW control design is suggested in this session. Kalman filter is first designed for two reasons: 1) it can remove the effect of white noise in PES, 2) state space description used for Kalman filter design can be extended to suggest a generalized time-domain control design.

In self servo track writing process, the only measurable signal in each track is position error signal which is the error between the current head position and the previously-written servo track. This PES is relative error between two servo tracks but not an absolute error from the track that should be written - a concentric circle with the same distances from other circles; this is one cause of the error propagation characteristics.

For this reason, the absolute error is needed to be estimated in order to suppress error propagation. Kalman filter can improve precision of absolute head estimation.

A. Estimation by Linear Addition of PES

Equation (14) is an algorithm to estimate the absolute position head suggested in [3].

$$e_n[k] = y_{n-1}[k] - y_n[k] + \frac{1}{1 + P}ξ_n[k]$$  \hspace{1cm} (13)

$$\sum_{i=1}^{n} e_i[k] = y_0[k] - y_n[k] + \sum_{i=1}^{n} \left( \frac{1}{1 + P} \right)^i ξ_i[k]$$

$$\Delta \hat{y}_n[k] = u_{nf}^{ff}[k]$$  \hspace{1cm} (14)

This suggestion is another interpretation of the reference correction which sets $F$ in Figure 2 as 1.

Since each PES has information on its own error from the previous servo track, the absolute error of current head position can be estimated by accumulating PESs in the previous tracks. Then, the estimated absolute error $Δ\hat{y}_n[k]$ is used as a reference correction to the next servo track writing, which will compensate the absolute error.

This estimation, however, also accumulates the measurement noise which increases $3σ$ of $Y$ and PES ([3]) and cannot completely prevent the error propagation due to the noise ([12]).

B. Effect of Measurement Noise in Head Position Estimation

With the feedforward control of Equation (14), the error to be attenuated will be $e_n[k] + u_{nf}^{ff}[k]$ in Equation (15)

$$e_n[k] + u_{nf}^{ff}[k] = y_0[k] - y_n[k] + \sum_{i=1}^{n} \left( \frac{1}{1 + P} \right)^i ξ_i[k]$$  \hspace{1cm} (15)

As this is the estimated absolute track squeeze error, the head position of $(n+1)$th track can be positioned on the correct circle like Equation (16) in the bandwidth of feedback controller.

$$y_n[k] \rightarrow y_0[k] + \sum_{i=1}^{n} \left( \frac{1}{1 + P} \right)^i ξ_i[k]$$  \hspace{1cm} (16)

The problem is the second term; the accumulation of the measurement noise. Even though the mean value of the measurement noise is zero, the variance increases in this accumulation. This is the propagation of measurement noise. Simulation indicates this characteristic.
C. Accurate Head Position Estimation by Kalman Filter Design

To remove the effect of the measurement noise, a Kalman filter is designed for correct estimation of the absolute head position. The dynamics from $y_n$ to $y_{n+1}$ is used for the dynamics of the Kalman filter design. VCM is modeled as a second order nominal model and the notch filters in the feedback controller are not included in this dynamics for simplification.

The states $x_n$ in Equation (17) has 5 states; 2 from the VCM and 3 from the PID controller.

$$
\begin{align*}
x_n[k+1] &= Ax_n[k] + Bu_n[k] + Gw_n[k] \\
y_n[k] &= Cx_n[k] + Bu_n[k] + Hw_n[k] + v_n[k]
\end{align*}
$$

The input $u_n$ is the signal to the PID controller. Since the reference position can be assumed to be 0 ignoring the offset between the write element and read element of the head, the reference correction $u^{ff}_n[k]$ will be this input $u_n$. The measurable signal PES is used as the output $y_n[k]$ of the dynamics. The output noise $v$ and the system noise $w$ are introduced to design the Kalman filter gain. The measurement noise can be taken into consideration using these two kinds of noise.

Considering these relationships, the Kalman filter is designed as Equation (18). The estimated relative error $\hat{e}_n[k]$ is accumulated for the reference correction in the next track.

$$
\begin{align*}
\hat{x}_n[k+1] &= A\hat{x}_n[k] + Bu^{ff}_n[k] + L[\text{PES}_n[k] + u^{ff}_n[k] - C\hat{x}_n[k] - Du^{ff}_n[k]] \\
\hat{e}_n[k] &= \hat{g}_n[k] = Cx_n[k] + Du^{ff}_n[k]
\end{align*}
$$

$L$ is the Kalman filter gain designed based on the covariance of $w$ and $v$ in Equation (17). Note that the measurement noise $\xi$ works not only as the output noise $v$ but also the system noise $w$.

In Kalman filter design, $v$ does not affects the states but it only affects the measurement $\xi$, however, affects the states $x$, since it is fed back to the feedback controller. Considering this point, one of the columns in $G$ in Equation (17) is designed as the same with $B$.

Figure 7 to 9 are simulation results of Kalman filter. In order to explore the effectiveness of Kalman filter to $\xi$, simplified simulations are conducted. Flutter Noise and RRO Noise are removed in these simulations and the measurement noise and torque disturbance are applied.

The reference correction filter FF changes from the gain 1 to 0.88. Figure 7 and 8 are comparison between the result of the gain 1 and 0.88. The gain 1 shows the best propagation suppression performance with all the disturbance including Flutter Noise and RRO Noise. However, when those disturbances are removed, the gain 0.88 shows the best performance.

This change in the best gain can be explained by the frequency characteristics of disturbances. The removed disturbances are in the high frequency range, which shows that higher gain is necessary to suppress the error propagation in the high frequency range. On the other hand, for the disturbances in the low frequency range such as measurement noise and
torque disturbance, the lower gain near 0.88 becomes the best gain. Figure 7 to 9 verifies this point.

Figure 9 is the result with the Kalman filter and the gain 0.88. Compared with Figure 9, the error propagation is suppressed more. Design of $K$ should be more explored to obtain better suppression performance.

D. Generalized Time-domain Design of SSTW Control

This Kalman filter design and the design of the controller $F$ in the previous section III are approaches to the SSTW control design based on time-domain system description, which distinguishes the suggested design from other SSTW control design based on Iterative Learning Control (ILC) where controllers are designed from an iteration-domain viewpoint using Markov parameters as system description.

The two approaches proposed in this paper can be stated on a same system description, and $F$ in Figure 2 can be designed in a more systematic way.

Using the states in each track described in Equation (17), the states of the extended system can be written like the following:

$$
\begin{pmatrix}
    x_n[k+1] \\
    x_{n+1}[k+1]
\end{pmatrix}
= \begin{pmatrix}
    A & 0 \\
    0 & A
\end{pmatrix}
\begin{pmatrix}
    x_n[k+1] \\
    x_{n+1}[k+1]
\end{pmatrix}
+ \begin{pmatrix}
    0 \\
    B
\end{pmatrix} u_{n+1}[k] + \begin{pmatrix}
    B \\
    0
\end{pmatrix} u_n[k]
$$

$$
c_n[k] = C(x_{n+1}[k] - x_n[k])
$$

Noise is not considered in this description for simplicity. Note that the input to control the system states is $u_{n+1}[k]$ which is used for reference correction. The output which is measured and used for design of $u_{n+1}[k]$ is $c_n[k]$. $u_n[k]$ cannot be designed in the dynamics of $X_n$ and only used as a pre-determined input signal. This consideration allows us to use the following notation.

$$
X_n = \begin{pmatrix}
    x_n \\
    x_{n+1}
\end{pmatrix}, U_n = u_{n+1}, Y_n = c_n, R_n = u_n
$$

With this notation, Equation (20) is re-described like the following:

$$
X_n[k+1] = A_E X_n[k] + \begin{pmatrix}
    B \\
    0
\end{pmatrix} R_n[k] + \begin{pmatrix}
    0 \\
    B
\end{pmatrix} U_n[k]
$$

This extended state description allows us to apply a variety of linear control theory to the SSTW system and design controllers in time domain.

Detailed control design and its comparison with the iteration-domain ILC design are to be researched.

VI. Conclusion

This paper designed controller for SSTW based on the time-domain system description. The propagation characteristics of SSTW play the most significant role in this time-domain control design. Several solutions to deal with the propagation problem were suggested: a reference correction design based on the analysis of the propagation characteristic in the frequency domain and accurate estimation of the absolute head position based on Kalman filter design.

State-space system description which can be used to design the suggested two controllers in a unified form was suggested as a general form of time-domain SSTW control.

As for the estimation of the absolute head position design, the simulation results showed the effectiveness of Kalman filter design on the measurement noise. The proposed reference correction based on that propagation characteristic succeeded to eliminate the error propagation and control the amplitude of the converged error.

Decrease in the converged error amount was not so large that more effective control design is required. This paper suggested a platform that can be used this effective control design. Actual design based on the suggested system description is future work.

REFERENCES